Reconceptualising Agency in a Senior Mathematics Classroom

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This paper explores the role of a problem-centred pedagogy (Collective Argumentation) in providing teachers with practices and tools to implement a problem-centred approach to teaching mathematics in a senior classroom. The paper focuses on a Year 11 lesson that required students to model and communicate the effects of a disease on a body's immune system. Using transcripts of teachers' talk the paper provides teacher perceptions of how students draw on the practices of mathematicians to illuminate and to communicate understanding to others. The study reveals that teachers consider that student capacity to act within a problem-centred curriculum, their sense of agency, is enhanced through participating in the practices of Collective Argumentation.

A view often held by the general community is that mathematics is about learning content. Coupled with this view is the notion that content is fixed and defined by what is developed in the text book, the answers to all problems explored are known and can also be found in the text book (Smith, 1996). Subsequently, the pedagogy adopted by many teachers is one of 'telling' the students the concepts they 'should' learn (van Oers, 1998). To reinforce the learning of content, the teacher 'tells' students by showing a worked example, by providing little tricks to make accessing the answer easier and then having the student practise the particular procedure on similar problems. If students do not understand, the teacher 'retells' the procedure and the students continue to practise (Smith, 1996). As a consequence, some students develop a rigid view of mathematics (Schoenfeld, 1988) where mathematics is seen as a discipline that has no real use apart from solving problems in a text book in a classroom setting. Not surprisingly, some students become disengaged with learning mathematics; they do not see mathematics as having any value or use outside the classroom and they endure their mathematics classes.

Ashton defines a teacher's "sense of efficacy" as a "belief in their ability to have a positive effect on student learning" (Ashton, 1985, p. 142) and that teachers build this sense of efficacy using "perceived past successes" (Smith, 1996, p. 389). Given this, the types of experiences the teacher has had in trying different pedagogies will impact not only on their willingness to try something new, but also on their perseverance in the face of adversity; be that adversity from students and parents in the way the concepts are being developed; adversity from administration in the form of a lack of provision of resources or a lack of professional development; adversity from colleagues that can be manifested in a variety of forms. Smith views a teacher's "sense of efficacy [as] more appropriately understood as a fluid, dynamic set of beliefs than a fixed personality trait" (Smith, 1996, p. 389). Hence, given the right set of circumstances: appropriate support, providing adequate professional development and resources, teachers are able to modify, even change significantly their "sense of efficacy" when teaching mathematics.

Changing the face of mathematics in Queensland schools is the Queensland Mathematics B Syllabus (Queensland Studies Authority, 2008) that identifies a number of key competencies developed in students as a result of completing the course of study. These competencies include "Collecting, analysing and organising information, communicating ideas and information, planning and organising activities, working with others and in teams, using mathematical ideas and techniques, solving problems and using technology" (QSA, 2008, p. 2). Given the emphasis on the development, justification and

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communication of mathematical ideas, the question then becomes “What may be done to provide teachers with the support to consolidate or refine their sense of efficacy; to be able to develop courses of study enriched though conjecture and exploration; that will engage students in learning mathematics; and that will develop the skill set proposed by the Queensland Studies Authority so as to ensure students will be able to participate in the 21st Century as informed citizens?”

Wilhelm, Baker and Dube (2001) suggest that a teaching model based on a Sociocultural approach to learning and development could be suitable for addressing many of the above curriculum requirements. That is, a model centred on teaching and learning relationships as opposed to a purely curriculum centred or solely student centred model. In such a model, students and expert others work together on tasks to develop solutions to problems. It is not the case of the teacher leading the discussion by ‘telling’ the students the mathematical concepts and then practising those skills many times, nor is it the case of a student blindly blundering on in the hope that at some point in the future they may build a relevant mathematical concept or understanding. Rather, it is a “community of learner’s model” (Wilhelm, Baker and Dube, 2001, p. 7) where an authentic problem is posed and both student and expert other, usually a teacher, but not necessarily so, work together to construct solutions to problems. The teacher intercedes as is necessary to ensure the student has the appropriate language to discuss the understandings being developed, ensuring the necessary mathematical concepts and procedures are being developed, consolidated or refined as is necessary and ensuring the student is able to make progress. The problems need to be of sufficient complexity to ensure challenge for the student, but not so complex that the student is unable to successfully complete it with assistance. (Wilhelm, Baker and Dube, 2001). One example of such a Sociocultural model of teaching and learning is Collective Argumentation (Brown & Renshaw, 2000).

Collective Argumentation

Collective Argumentation is an approach to teaching and learning that is based on five interactive principles necessary for students to engage with the task and each other in order to ensure successful outcomes for each student in the class. The first principle, the ‘generalisability’ principle (Brown, 2005), requires that students stop and think about the problem that has been posed. The student should think about the problem in terms of what mathematical concepts and procedures might be useful in building a solution. Students are encouraged to make links with prior knowledge, procedures and understandings. Initially these links may not be strong but as the discussion with the other members of the group and the teacher occur in this and later stages of the process there are opportunities for these links to be strengthened.

During this stage, the teacher could choose to work with less able students, discussing ideas and building on suggestions they make so that they have something to bring to the sharing table for discussion. It may be that the whole class might experience difficulty with the task that has been set, so the teacher may decide to engage in a whole class discussion that provides students with assistance in understanding the context the task and what is required.

It is here that the teacher may ask each student to submit their work to that point. The teacher is then able to use this information to ascertain what understandings each student might have at that moment in time in terms of mathematical content, procedures, and ideas. Armed with this information the teacher, can the plan learning experiences, either immediate or future, that could be used to remediate misunderstandings, or used to extend and enhance the understandings of an individual student, small groups of students, or the whole class.

The second principle, the ‘objectivity’ principle, requires that ideas, relevant to the task are objectified and communicated to other members of the group. All students having engaged in the ‘generalisability’ principle are able to share their ideas. These may have been co-constructed with the teacher, but none the less, each idea remains on the table until it can be rejected. It may only be rejected if it can be denied by reference to past experiences or logical-mathematical reasoning. If the ideas cannot be denied then they must remain part of the discussion. Third, the ‘consistency’ principle requires that ideas which are contradictory to each other or that belong to mutually exclusive points of view must be resolved through discussion.

During the second and third stages of the Collective Argumentation process, the teacher is able to listen to the statements being made by students in their groups and provide input when inaccurate statements are made or conceptual misunderstandings are identified. The teacher is able to work with individuals or small groups to correct these inaccuracies and conceptual errors and the student or group of students are able to use these developing ideas in the subsequent discussion and development of a solution to the problem.

The fourth principle is ‘consensus’. Consensus requires that all members of the group understand the agreed approach to solving the problem. If a member of the group does not understand, there is an obligation on that student to seek clarification, and a reciprocal obligation on the other group members to assist.

Finally, in implementing the fifth principle, ‘recontextualisation’, students re-present the group response to the other members of the class for discussion and validation. Communicating to class members outside the group, challenges students to rephrase, defend, and to reassess their thinking. This is an important part in the model. It is here that the teacher and the rest of the class get to view the understandings of individuals within the group and the understanding the whole group, as the members of the group share their analysis of the task and their subsequent synthesis of ideas, content and procedures. In making their understanding public, students as a group will need to be able to justify their conclusions and defend those conclusions as other members of the class ask probing questions.

This paper focuses on the different principles of Collective Argumentation for the purpose of exploring how a class of Year 11 students drew on the practices of Collective Argumentation to illuminate and to communicate understanding to others.

Method

Lesson Context. The lesson, upon which the data used in this paper is based, was situated in a Year 11 Mathematics B lesson. The class consisted of 13 Year 10 students. As they were gifted students of Mathematics, they were accelerated one year in their study of mathematics. This group of students had been studying mathematics together for the previous three years in this accelerated class.

Teacher Reporting Context. During a professional development session on Collective Argumentation the teacher, Jill, presented the activity she conducted during this lesson to other teachers and mathematics educators and shared her reflections on teaching the lesson. The professional development session was a part of a larger study into teachers’ appropriation of the practices of Collective Argumentation into their everyday teaching of mathematics and/or science. The larger study, conducted over a three-year time frame, involved university educators working with 20 school teachers of mathematics and/or science from 6 schools located in South-East Queensland to bring about and reflect upon change in the way they teach mathematics and/or science.

Research participant. Jill is a teacher with significant experience in the classroom teaching mathematics. Having taught in a number of different educational jurisdictions both in Australia and overseas, Jill was based in a middle-class Independent College situated in a major city.

The task (see Figure 1) that was the focus of Jill's report comprised a data set taken from the website http://www.tbiomed.com/content/3/1/10.

John is attending a local hospital to receive chemotherapy treatment for leukaemia. A guideline suggested for chemotherapy patients is; they may be discharged when their White Blood Cell (WBC) count has stabilised.
Using the data in the table below, determine when John may return home. You must justify your decision. (Normal WBC count is between 4000-10000.)

Day	White Blood Cell Count (thousands) WBC
5th September	2.51
1	2.11
2	1.71
3	1.35
4	1.18 ***(Stem cells introduced today.)***
5	0.82
6	0.75
7	0.5
8	0.4
9	0.45
10	0.45
11	0
12	0.47
13	0.57
14	0.75
15	0.82
16	0.95
17	1.18
18	1.41
19	1.63
20	2.02
21	2.3
22	2.8
23	3.35
24	3.5
29	4
32	4.4
38	4.2

Figure 1. The Task.

Analysis

Jill's report was transcribed for analysis. The analysis employed a form of Discourse analysis that focused on the different principles upon which Collective Argumentation is based: *Generalisability*, *Objectivity*, *Consistency*, *Consensus*, and *Recontextualisation*. According to her report (see Table 1), Jill commenced the task (see Figure 1) by asking the students to use Collective Argumentation to develop and explore their own ideas individually. This allowed them time to reflect and attempt to identify links with their current knowledge, procedures, and understandings. During this time Jill was able to work with individuals to identify where their thinking was taking them and what strategies they might be considering.

Table 1
Generalising ideas

Turn	Text
58	…………………………………., I give
59	them about five or six minutes to, just to sit on their own
60	and think about what they need to do and consider what
61	they've done previously, what they can bring (to the task).

During the 'Objectivity Principle' stage of the Collective Argumentation process, Jill reported that she encouraged students to question each other and engage in mathematical discussions to identify possible strategies. As the groups of students consolidated their strategy as to how best to proceed, Jill reported that there were two different tacts taken (See Table 2, lines 67 through to 69); some groups decided to model the data using a quadratic equation, while other groups decided to model the data using a cubic equation.

Table 2
Objectifying ideas

Turn	Text
61	…………………………………………………..Half of
62	them strangely decided to model it as a quadratic, half of
63	them decided to model it as a cubic. And of course my
64	response is 'oh okay'. Give nothing away, cause the kids
65	need to say 'well that makes sense to me', it makes sense
66	given the scatter plot that it is, given that I don't know
67	much about the context, so we go forward. So half of them
68	modelled it as a quadratic as you can see there (points to an example of student work), half of them
69	modelled it as a cubic, …

This sharing of her practice is very telling about Jill's values and about how she works with the students in her class to learn mathematics. Her comment in line 64 ".. okay. Give nothing away..", indicates that while Jill was there to support students, she was not going to intercede in the strategy students were going to explore. Instead, Jill would try to ensure that the students developed the appropriate mathematical techniques and procedures for their method of solution, developed and used the appropriate mathematical language, but the students needed to develop the solution. If it turned out to be an inappropriate strategy that the students were developing, then that was going to be as valuable a learning experience as if their strategy did develop valid conclusions.

During the 'Consistency' and 'Consensus' stages of the Collective Argumentation process, some groups overheard other groups speaking about their choice of model – quadratic equation versus a cubic equation. This 'overhearing', according to Jill, created a need within groups to mathematically justify their choice of model (see Table 3).

Table 3
Making ideas consistent

Turn	Text
80	a couple of them had actually said to me, I'm looking at the
81	gradient function and it's not helping, and I said well okay
82	what are you going to do? Two of the kids had decided in
83	their groups, 'well it (the function) doesn't tell me that it's true, that it's
84	correct, but it doesn't tell me that I'm wrong, so I'm going
85	to do it anyway'. Of course one of them (a group) had done cubics
86	and one of them had done quadratics, so still no firm
87	decision. So of course then they were exploring the context,

In this situation, Jill reported that the gradient function provided no help, so the students went back to their notes and decided to reconsider the context and the attributes of each of the models to see if one made more sense than the other. In Table 3 lines 84 and 85, Jill reported that the students decided that either equation would be a suitable model.

Table 4
Coming to a consensus

Turn	Text
99	……………………………………there wasn't really much
100	difference between the cubic or the quadratic, except above
101	the line [equal to the 4000 WBC count] which we said you were okay. So what the kids
102	decided, what they decided as a group was we should
103	probably model it right up until you were okay. …

In Table 4 we see the students entering the 'Consensus' stage. Jill recounts in lines 99 through to 102 that as students engaged in the modelling process, they needed to concern themselves not only with the suitability of the equation in terms of how well it modelled the data provided, but also in its ability to allow reliable predictions to be made outside of the data set. In considering the suitability of the model, Jill reported that the groups of students who modelled the relationship using the cubic function were concerned that their model would not be useful outside the given data. In line 103, Jill reports that the group decided that it was important for the model to be applicable until "…you were okay", that is, the patient was healed.

Table 5
Challenging the consensus- the Recontextualisation phase

Turn	Text
118	…………………. one of the
119	other kids from the room said well if we use the cubic model
120	we would predict that every person who had the treatment
121	would actually get sick again…..

During the Recontextualisation phase, Jill reports that groups of students were asked to present their strategies and to validate and justify their conclusions. As students challenged each other to justify their models, Jill reported that a whole class discussion developed which considered the task and what could be done. In Table 5 Jill reports that the students were concerned that the cubic model predicted that every person became ill again. The students were attempting to use their model to interpret the context outside of the data set. Jill went on to share that, the class identified the need to establish limitations for their models and hence the students identified suitable domains for their equations. Jill further indicated that the class explored the notion of building two separate functions for the data set. This produced piecewise functions with the domains of $0 \le x \le 4000$ and $x \ge 4000$.

Discussion

Through the implementation of Collective Argumentation, Jill allowed students to make their thinking visible. Jill's "sense of efficacy" is made manifest through her choice of data from a context that was interesting and had meaning in the world outside the classroom; data that was a catalyst for students to explore, to further develop and communicate their mathematical understandings. Through an analysis of the context and data, students needed to make conjectures about the types of models that may be used to model the data. With a list of potential models, students were required to explore them in order to provide justification for their conclusions. Having decided on the most suitable model, students then needed to consider the limitations of their equation and investigate whether it adequately modelled the context outside the data set.

Using the principles of Collective Argumentation, Jill's "sense of efficacy" allowed her to encourage her students to implement the procedures a mathematician would employ to solve a problem of this type. During the "Generalisability" stage students were encouraged to identify from within their own knowledge set, concepts that could be useful in the modelling of the data. In the "Objectifying" stage students shared their ideas and provided justification as to why their models should be considered. Evidence from the "Consistency" and the "Consensus" stages, indicated that students were challenged by each other and by Jill to provide justification for their choice of model before proceeding. Students were forced to reconsider the context in light of the data and unable to obtain any guidance from either the context or the mathematical tests, students had to make decisions about the models they were exploring. As a consequence of these deliberations, the groups of students decided to proceed with building their respective models. During the "Recontextualisation" stage, students shared and defended their models and considered that perhaps no one model adequately modelled all of the relationships exhibited by the data from the context. Indeed it was decided that it might be better to develop a piece-wise model, to model the different sections of data.

Jill's "sense of efficacy" is evidenced through her willingness to hold back and give students a space to think, ".. okay. Give nothing away, [be]cause the kids need to say well that makes sense to me,…" Jill encouraged her students to be thoughtful about the context and to build meaningful insights. As the students worked through their strategy, Jill was able to interact with her students and make sure they were building appropriate understandings of the concepts and encouraged the students to communicate and justify those understandings. During the process, students considered a number of different viewpoints, each viewpoint having strengths that needed to be acknowledged and limitations that needed to be identified and dealt with. It was not the case of the teacher or the text book 'telling' the students their answer was right, rather it was the students' understandings of the mathematics and its ability to model the context that gave meaning to any conclusions they derived. As the students worked collaboratively they were able to build on each other's understanding and develop their knowledge to quite advanced levels.

It can be seen from the above analysis that Jill's 'sense of efficacy' in this lesson was not based on what 'should' be completed by students, but on what 'could' be attempted (Van Oers, 1998) through the incorporation of Collective Argumentation into the teaching and learning of her classroom. For Jill, building, using, and understanding the mathematics was the focus of the task. The context allowed students to see the value of the mathematics they were using and through this use of mathematics, the students were encouraged to realise that the mathematics they use in the class has valuable application outside the classroom.

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